Determination of Dielectric Constant
Of Printed Circuit Boards

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Introduction

The dielectric constant of PCB material is important in determining the trace width required to produce a characteristic impedance of 50 ohms, or any other desired impedance. The most commonly used material, FR-4, has a dielectric constant that may vary widely between manufacturers or batches, and also varies over frequency. It is useful to have a method to determine the dielectric constant for a particular board in order to properly use that board. We here present a largely non-destructive way to measure the dielectric constant for a particular board.

Technically, we are seeking the *relative* dielectric constant (ε_r); i.e. the value relative to the dielectric constant of a vacuum. But we here just refer to it as the dielectric constant (ε).

References

The method utilized here was suggested by two sources:

1. Howell, "A Quick Accurate Method to Measure the Dielectric Constant of Microwave Integrated-Circuit Substrates", *IEEE Transactions on Microwave Theory and Techniques*, March, 1973.
2. Wang, "Determining Dielectric Constant and Loss-Tangent in FR-4", *UMR EMC Laboratory Technical Report TR-00-1-041*, March, 2000.

These articles each discuss methods of treating the substrate as a resonant cavity, and determining dielectric constant from the resonant frequencies of such cavities. They are referred to below as "Howell" and "Wang". Neither uses exactly the technique described here.

Cavity Math

In an essentially two-dimensional cavity with length and width but very small height, many resonances are possible. The simplest resonances are at frequencies where the half-wavelength equals the length or the height, but there are many other possibilities. Each resonance corresponds to an integral "wave number" or "mode number" in each direction (length and width). If p is the mode number in the length (L) direction and q is the mode number in the width (W) direction, then the resonant frequency for a given p and q is:

$$f = \frac{c}{2\pi\sqrt{u\varepsilon}}\sqrt{\left(\frac{\pi \cdot p}{L}\right)^2 + \left(\frac{\pi \cdot q}{W}\right)^2} \quad \text{(Eq. 1)}$$

Where c is the speed of light, μ is permeability (for PCB dielectrics, this is 1), and ε (epsilon) is the dielectric constant. Each of p and q can have any non-negative integral value, except both cannot simultaneously be zero.

To measure L and W in inches, and f in MHz, we can convert the above formula into this form:

$$f = \frac{983.6}{2\pi\sqrt{\varepsilon}}\sqrt{\left(\frac{12\pi \cdot p}{L}\right)^2 + \left(\frac{12\pi \cdot q}{W}\right)^2} \quad \text{(Eq. 2) (f in MHz, L and W in inches)}$$

If we measure the resonant frequency f for a given p and q, we can calculate the dielectric constant as:

$$\varepsilon = \left(\frac{983.6}{2f}\right)^2\left[\left(\frac{12 \cdot p}{L}\right)^2 + \left(\frac{12 \cdot q}{W}\right)^2\right] \quad \text{(Eq. 3) (f in MHz, L and W in inches)}$$

(Note: the speed of light is 983.6 million feet per second.)

Application to Printed Circuit Boards
A double-sided PCB might be treated as an open-sided cavity, so for the moment assume that it is proper to do so. If one side is treated as the ground plane, the signal can be loosely coupled to the other side, creating a signal that travels primarily between the two planes, inside the PCB dielectric. If we measure certain resonant frequencies of this cavity, we can calculate the dielectric constant from Eq 3. We first do this in transmission mode. A piece of double-sided FR-4 is cut to 4" x 5". SMA connectors are soldered in two opposite corners. They can be soldered to one side with their legs broken off and the center pin extending through a hole to the other side, but not actually soldered to the other side. Or, for thin material, they can be mounted edge-wise with the center pin projecting over but not touching the top copper plane. The stimulus signal is attached to one connector and the response is taken from the other. Figure 1 shows the test setup (attenuators are optional).

Figure 1—PCB Transmission Test Setup
For FR-4, the connectors were vertically mounted without legs, with the center pin projecting through the board but not attached to the other side.

Figure 2 shows the results.

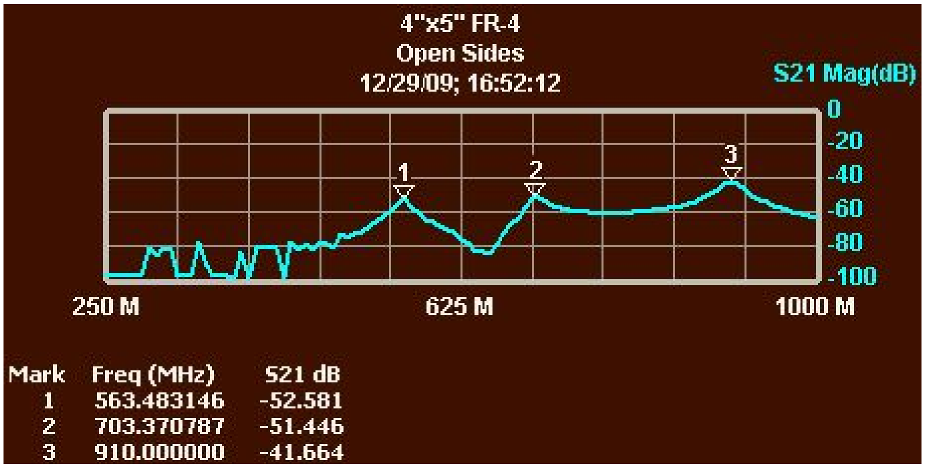


Figure 2—Transmission of FR-4 from corner to corner

There are three resonances shown in Figure 2. Marker 1 shows the resonant frequency for which the half-wavelength is the longer dimension. Marker 2 shows the resonant frequency for which the half-wavelength is the shorter dimension. If we treat length as the longer dimension, and label the modes as M(p,q), then marker 1 is M(1,0) and marker 2 is M(0,1). Marker 3 is the first mode that involves resonance occurring in both directions and is M(1,1).

This notation for modes differs from the typical notation TE_{101}, etc. For our purposes, all we care is that each mode is identified by two integers, p and q, which integers we can plug into Eq. 3. We are not concerned with the details of the signal propagation.

If we apply Eq. 3 to each of these resonances, using the f, p and q values for each, we get a dielectric constant of 4.4 for markers 1 and 2 and 4.3 for marker 3. Those are very plausible values for FR-4, though its dielectric constant can vary so much between boards that we can't make any judgment about the accuracy of our method from this data.

Instead of measuring transmission, we can place a connector in the middle of the board, with the body soldered to one side and the center pin extending through a hole and soldered on the other side, and then measure the reflection at this connector. Figure 3 shows the results.

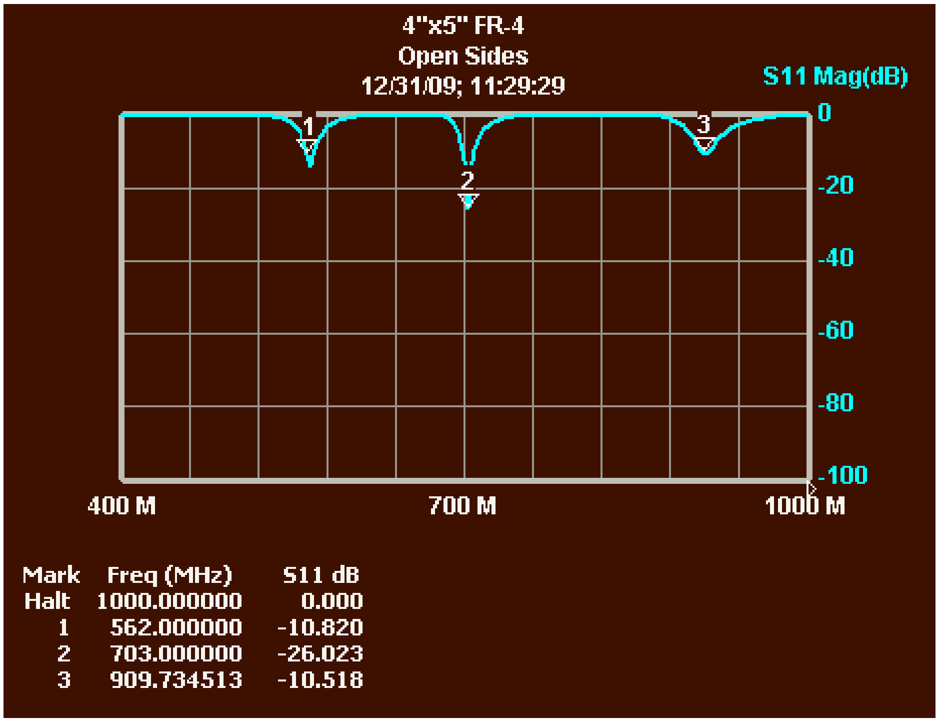


Figure 3—Reflection of PCB with connector in the center

The resonances shown in Figure 3 are the same as those in Figure 2, so either method is suitable to locate the resonances.

We have so far treated the PCB as a cavity, even though the sides are open. If we seal the sides of the board with copper tape, we will in fact have a closed cavity. While Howell indicates that we should be able to measure transmission of this cavity, we got no meaningful response when measuring the corner-to-corner transmission of such a closed cavity. We simply couldn't come up with a good method to get the signal inside the cavity. With the connector described above for measurement of reflection, we can see a couple of resonances up to 3 GHz, but none that seem consistent with the above equations.

So we will stick with the open-sided PCB. It might be questioned whether this is truly acting as a cavity, or simply as a sheet of copper over a ground plane, but in fact we will see from measurements of GML 1000 that the two seem to be pretty much the same thing. Indeed, in several of the later measurements, grasping the PCB on both sides with fingers had minimal impact on the measurement, suggesting that all the "action" is occurring in the interior of the PCB.

Tests with GML 1000

GML 1000 was a precise PCB material that is no longer manufactured. Its dielectric constant was specified as 3.05 ± 0.05, at both 2.5 GHz and 10 GHz. We will assume the same spec applies below 2.5 GHz. This precise specification gives us a good reference for comparison of our own measurements.

A 5” x 4” piece of double-sided GML 1000 was fit with SMA connectors on two opposing corners. Each connector was mounted edge-wise, with two legs soldered to the bottom plane and the center pin extending over, but not attached to, the top copper plane. This provided loose coupling of the signal to the top plane. Transmission was then measured. This method has the advantage that no holes are drilled; other than a bit of solder in the two corners, the PCB is not harmed by the test.

Three separate scans were conducted in order to cover the range up to 3 GHz, using the 1G, 2G and 3G frequency bands of the MSA.

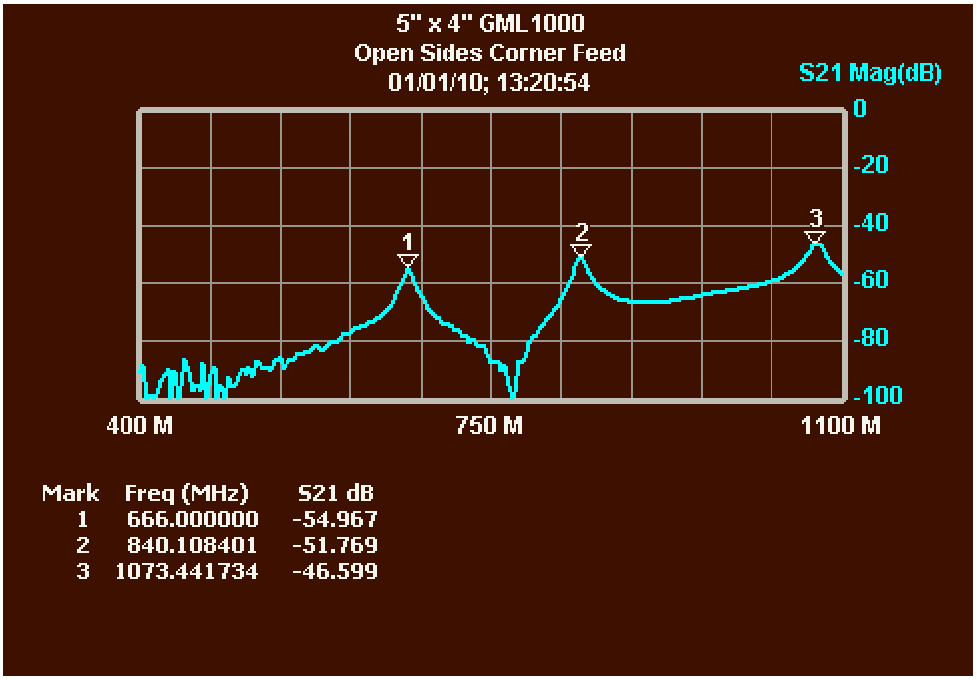


Figure 4—Low Frequency GML

Figure 4 shows the first 3 modes: M(1,0), M(0,1) and M(1,1), where the first mode number (p) refers to the longer dimension. Thus, M(1,0) represents the frequency with a half-wavelength equal to 5”. These three frequencies are always easy to identify, except that if the PCB is nearly square, resonances 1 and 2 will merge to a single peak.

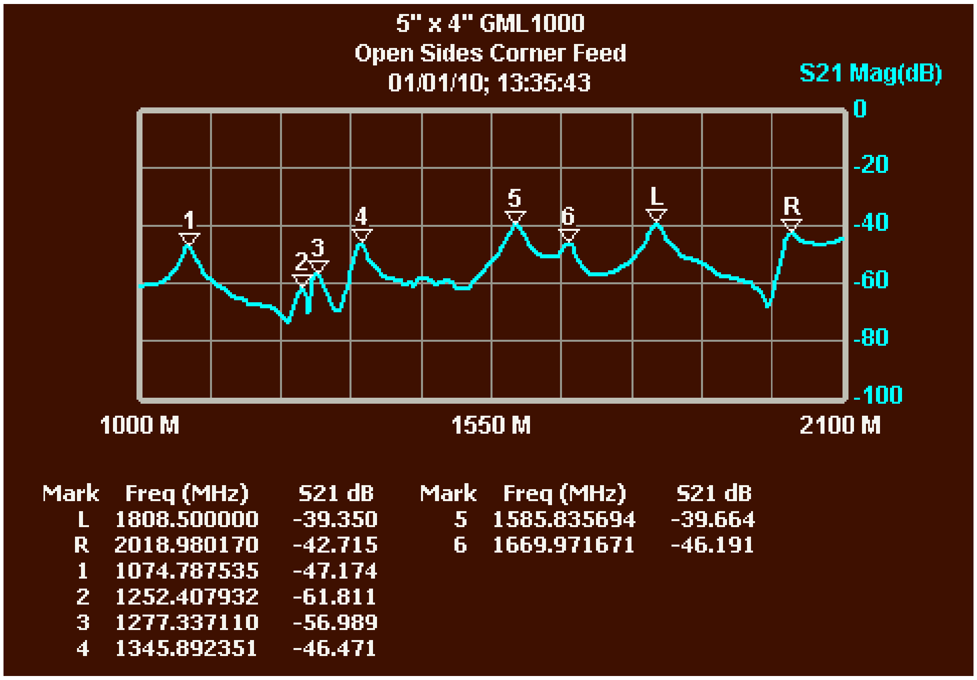


Figure 5—Mid Frequency GML

Figure 5 is taken with the MSA set to the 2G band, and shows many resonances, most of which will be matched to specific modes a bit later.

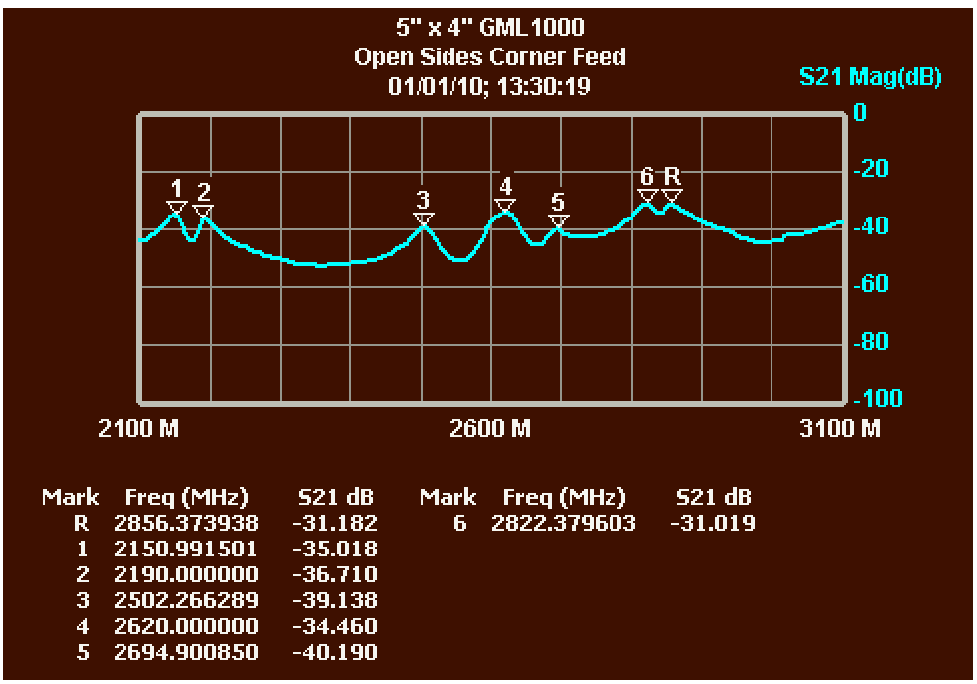


Figure 6—High Frequency GML

Figure 6 is taken with the MSA set to 3G band, and also contains many resonances.

To match the resonances to specific modes, we first calculate the dielectric constant from the first three resonances, which are easy to identify. They provide values of approximately 3.1, so we assume that as the actual value and calculate where all the resonance peaks should occur. We then match the actual peaks to the calculated peaks as best we can. The matches are shown in Table 1.

Table 1—Calculated and Actual Resonant Modes of 5" x 4" GML 1000

Assumed ε=	3.1	
Length=	5	in
Width=	4	in

Sorted by Mode

p	q	Calc Freq	Meas Freq	Calc ε
0	1	838	840	3.09
0	2	1677	1669	3.13
0	3	2515	2502	3.13
1	0	671	666	3.14
1	1	1074	1073	3.10
1	2	1806	1808	3.09
1	3	2603	2620	3.06
2	0	1341	1345	3.08
2	1	1582	1585	3.08
2	2	2147	2151	3.09
2	3	2851	2856	3.09
3	0	2012	2019	3.08
3	1	2180	2190	3.07
3	2	2619	2620	3.10

Sorted by Freq

p	q	Calc Freq	Meas Freq	Calc ε
1	0	671	666	3.14
0	1	838	840	3.09
1	1	1074	1073	3.10
2	0	1341	1345	3.08
2	1	1582	1585	3.08
0	2	1677	1669	3.13
1	2	1806	1808	3.09
3	0	2012	2019	3.08
2	2	2147	2151	3.09
3	1	2180	2190	3.07
0	3	2515	2502	3.13
1	3	2603	2620	3.06
3	2	2619	2620	3.10
2	3	2851	2856	3.09

The left part of the table lists the resonances by mode; the right side lists them by frequency. The left two columns in each part are the p and q values, p being the mode number for the Length dimension. The third column is the frequency at which the mode should appear if ε=3.1. This column is used to match the mode to a specific peak; the actual frequency of the matched peak is in the fourth column. Finally, from the measured frequency we calculate ε, which is listed in the final column of each table. The measured frequency 2620 MHz appears twice in the table, because it is ambiguous as to whether it matches M(1,3) or M(3,2); but either way the ε value calculated for that frequency is very good.

Table 1 shows that we can match most of the resonant peaks to specific resonant modes, and the calculated dielectric constants are very consistent, varying from 3.06 to 3.14. Recall that the actual value for GML 1000 is anywhere from 3.0 to 3.1. Our measurements are therefore probably about 1.5% high.

Analysis of Error

The error in the measured values of dielectric constant for GML 1000 are fairly small. The average value is about 3.1, which is within the spec value, but on the high end. There is a correction that can be made to Eq. 3, to adjust each measured frequency to account for effects of the finite Q of the resonance. That correction adjusts each resonant frequency upward, which has the effect of adjusting the calculated ε downward. We did not measure Q, but this adjustment likely would at most affect the calculated ε value by about 1%, though that adjustment would be in the right direction to reduce the likely error.

Another source of error is the measurement of the dimensions of the PCB. In this case, error of about 1% of ε might be due to such errors.

Howell suggests that measuring a substrate with open sides leads to error and erratic results due to radiation losses. However, our results much more self-consistent than the ones he cites for open-sided measurements. Efforts to measure transmission with closed-sided boards (sealed with tape) were not successful. Wang describes a method for doing so with FR-4 using reflection measurements, but his method of coupling the signal to the cavity created by sealing the board is not clear, and efforts to duplicate his work produced very few resonances and ε values that seemed 20-30% in error.

FR-4's Varying Dielectric Constant

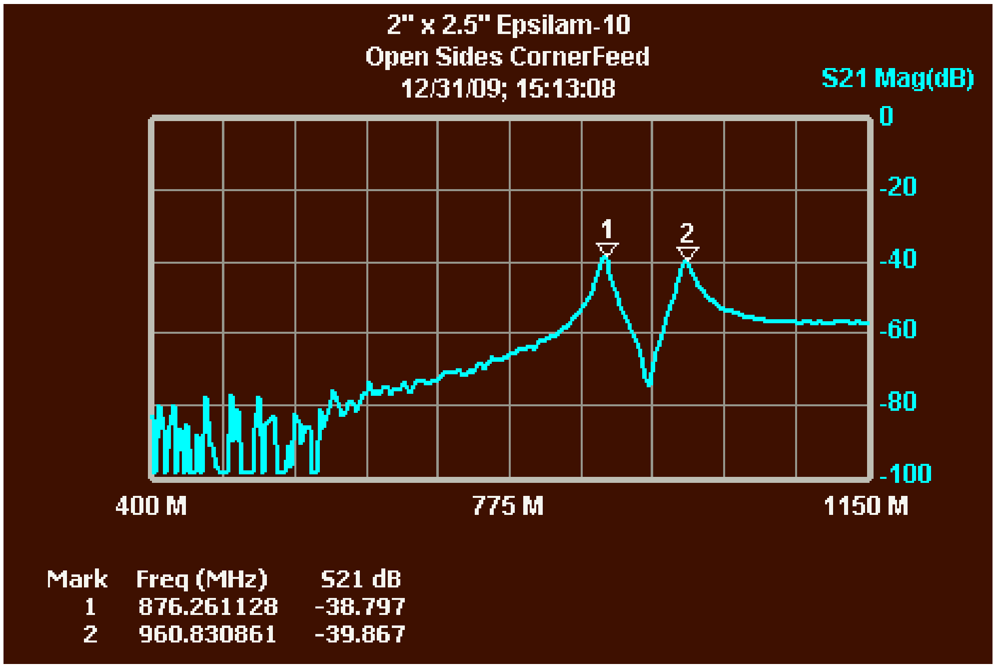
Applying the technique used in Table 1 to match resonances to specific modes is a little trickier for FR-4. Its dielectric constant varies with frequency, so calculating expected resonances from the ε value determined for the first several resonances may not provide sufficiently precise data to determine which resonance matches which mode. The assumed value may have to be tweaked to get good matches in the 2G and 3G ranges.

Conclusion

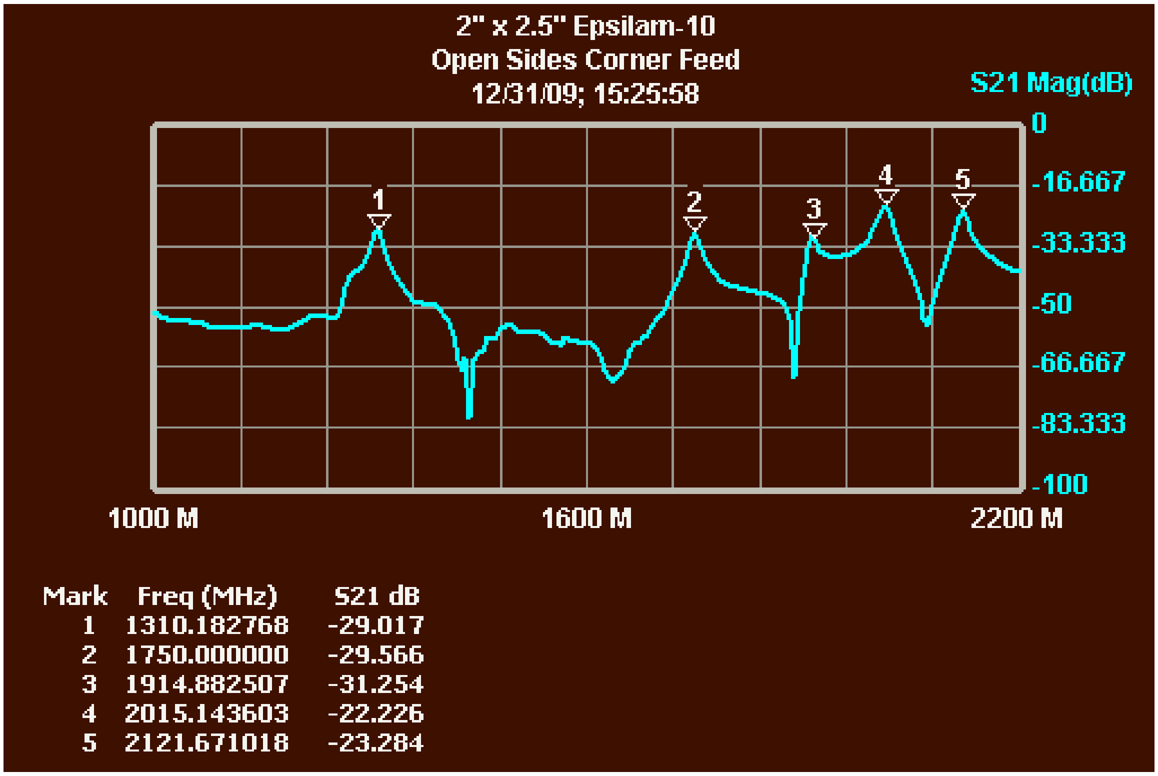
The described method of calculating the dielectric constant from resonant frequencies, using non-destructive transmission measurements, appears to provide accuracy of somewhere in the 2% range; we cannot pin it down precisely from our data. For some perspective, the characteristic impedance of a 50-ohm trace calculated for ε =3 will decrease by only one ohm if ε turns out to be 3.15, an error of 5% in ε.

APPENDIX—Tests with Epsilam-10

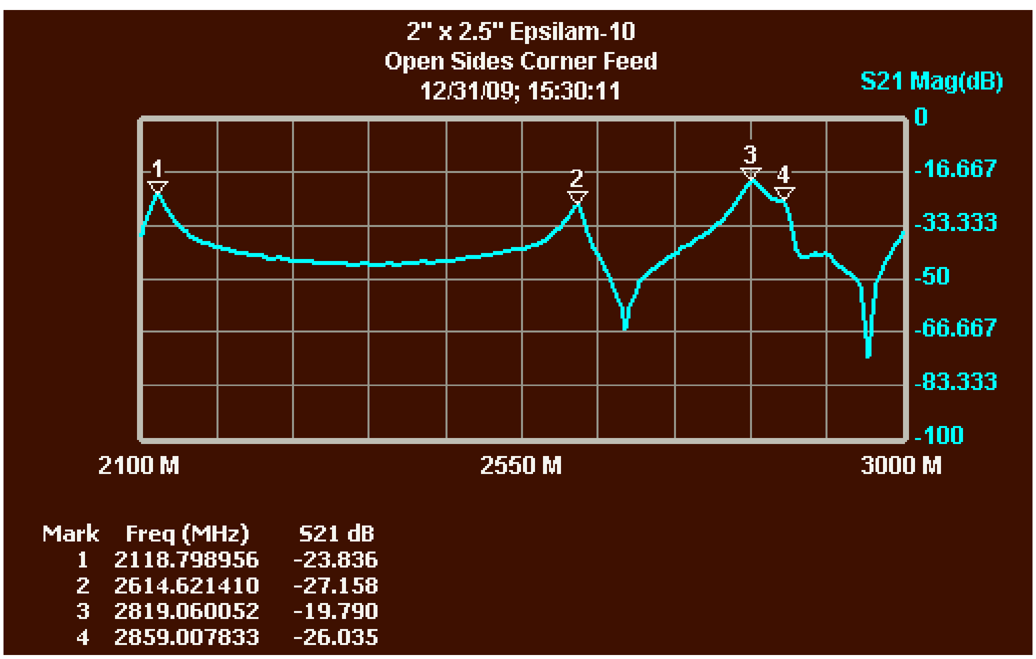
The following scans were done with a 2” x 2.18” piece of Epsilam-10, in the same manner as the tests described for GML 1000. (The graphs incorrectly state it is 2” x 2.5”.) There are three scans, using the 1G, 2G and 3G frequency bands of the MSA.



The first scan shows the two simplest resonant modes. If we let p=mode number for the 2.2” dimension and q=mode number for the 2” dimension, and label the modes M(p,q), then marker 1 shows M(1,0) and marker 2 shows M(0,1).



The second scan shows several more modes. Marker 1 is M(1,1). Markers 1, 2 and 4 are later matched to specific modes. Markers 3 and 5 are ambiguous.



The third scan shows the modes from 2100 MHz to 3000 MHz. Markers 2 and 3 are later matched to specific resonance modes.

Using the same approach as described in the main text, the resonances were matched to specific modes as shown in Table A-1.

Assumed e=	9.4	
Width=	2.18	in
Height=	2	in

p	q	Calc Freq	Meas Freq	Calc e
1	0	883	876	9.55
0	1	963	960	9.45
1	1	1307	1310	9.34
1	2	2119	2121	9.37
2	0	1767	1750	9.57
2	1	2012	2015	9.36
2	2	2614	2614	9.39
3	1	2820	2819	9.40

Table A-1

Using the measured frequency and the mode numbers, the dielectric constant was calculated in the final column. There is extremely good consistency among the measurements, all being in the range 9.45 ± 1%

The best information I could find on the dielectric constant of Epsilam-10 is that it is 10.2 ± 0.25. This makes the above measurements approximately 5-10% low.

My guess is that the error is due to the fact that the speed of propagation in the board is affected by the fact that the field near the board edges is partially in air, which makes the effective dielectric constant a blend of that of air (near 1) and that of the Epsilam-10. The extreme difference between those two values makes this effect stronger for this board than for the GML 1000 board, with a dielectric constant near 3. The smaller Epsilam board also has twice the edge length relative to it area, which also makes the dielectric constant of air more of a factor in the final blend.